Convergence of Series in Mittag-Leffler Type Functions

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Abstract. Explicit solutions of some kinds of fractional order (or multi-order) differential and integral equations involving Erdelyi-Kober (E-K) operators are representable by means of series in Mittag-Leffler type functions like those considered in this work. The domains of convergence of such expansions are found. The series behaviour on the boundary of these domains are studied. Cauchy-Hadamard, Abel, Tauber and Littlewood type theorems for such series are given. Asymptotic formulae for "large" values of indices of these functions, used in the proofs of the convergence theorems for the considered series, are also provided.

Keywords: Multi-index Mittag-Leffler functions, Cauchy-Hadamard, Abel, Tauber and Littlewood type theorems, summation of divergent series, asymptotic formula.
PACS: 02.30.Gp, 02.30.Lt

INTRODUCTION

The Mittag-Leffler (M-L) function, called the "Queen"- function of Fractional Calculus (FC) (in [4]), has also recently become a very popular and exploited Special Function of Fractional Calculus (SF of FC). It was introduced by Mittag-Leffler (1903-1905), studied and generalized to two indices also by Wiman (1905), Agarwal (1953), Humbert (1953), and a description of its basic properties appeared in the Bateman Project [3] ("Higher Transcendental Functions", Vol.3), in a chapter devoted to "miscellaneous functions", and later in Dzrbashjan's book [2]. Unfortunately, for many years it continued to be unknown to the majority of applied scientists and mathematicians, simply because it has been ignored in the common handbooks on (classical) special functions and in the tables of Laplace transforms, despite its applications. Back in 1931 Hille and Tamarkin provided a solution of the Abel integral equation of the second kind in terms of this function. The Queen era of the M-L function started recently, along with the revival of the classical FC, when it was recognized as providing solutions to fractional (arbitrary) order differential equations, modeling fractional order control systems [20], fractional viscoelastic models [13], continuous random walks - as the waiting time density corresponding to a master equation with fractional time derivative of order α and type $\beta > 0$ [5], for solving various fractional order differential and integral equations, and this stimulated fractional analysts to develop new analytical and numerical results for them. See also in: [[8], Appendix], Gorenflo and Mainardi [4], Kilbas, Srivastava and Trujillo [7]; many papers in the "FCAA" journal, etc. The Mittag-Leffler functions are natural "fractional index" $(\alpha > 0)$ extensions of the exponential function and trigonometric functions like cos-function. They are defined in the whole complex plane $\mathbb{C}$ by

CP1301, *Applications of Mathematics in Technical and Natural Sciences (AMiTaNS '10)*
edited by M. D. Todorov and C. I. Christov

the power series:

$$E_\alpha(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+1)},\quad E_{\alpha,\beta}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)},\qquad (\alpha>0,\ \beta>0). \tag{1}$$

The functions have been studied by Dzrbashjan [2]. The detailed properties of these functions can be found in the contemporary monographs [7], [8], and [20].

Recently a class of special functions of Mittag-Leffler type that are multi-index analogues of $E_{\alpha,\beta}(z)$ has been introduced and studied by Kiryakova (see, e.g., [9], [10]). The indices $\alpha := 1/\rho$, $\beta := \mu$ are replaced by two sets of multi-indices $\alpha \to (1/\rho_1, 1/\rho_2, ..., 1/\rho_m)$, and $\beta \to (\mu_1, \mu_2, ..., \mu_m)$.

Definition 1. Let $m > 1$ be an integer, $\rho_1, ..., \rho_m > 0$, $\mu_1, ..., \mu_m$ be arbitrary real (complex) numbers. By means of these "multi-indices", the multi-index Mittag- Leffler functions are defined as:

$$E_{\left(\frac{1}{\rho_i}\right),(\mu_i)}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma\left(\mu_1+\frac{k}{\rho_1}\right)\ ...\ \Gamma\left(\mu_m+\frac{k}{\rho_m}\right)}. \tag{2}$$

The same functions, considered also by Luchko [12] and Yakubovich and Luchko [22] are called Mittag-Leffler functions of vector index. As proved in [10], the multi-index Mittag- Leffler functions (2) are entire functions of order ρ with $\frac{1}{\rho}=\frac{1}{\rho_1}+...+\frac{1}{\rho_m}$ and type $\sigma=\left(\frac{\rho_1}{\rho}\right)^{\frac{\rho}{\rho_1}}...\left(\frac{\rho_m}{\rho}\right)^{\frac{\rho}{\rho_m}}$.

Explicit solutions of some kinds of fractional order (or multi-order) differential and integral equations involving Erdelyi-Kober (E-K) operators are representable by means of series in M-L type functions like those considered in this work. In this paper the domains of series convergence in similar kind of functions are found. The series behaviour are studied on the boundary of these complex domains. Cauchy-Hadamard, Abel, Tauber and Littlewood type theorems for such series are given. Asymptotic formulae for "large" values of indices of these functions, used in the proofs of the convergence theorems for the considered series, are also provided. In our previous papers ([15]-[17]) we studied series in systems of some SFs of FC which are fractional indices analogues of the Bessel functions and also multi-index M-L functions (in the sense of Kiryakova [11]).

EXAMPLES OF MULTI-INDEX MITTAG-LEFFLER FUNCTIONS

A special case (for $m \geq 2$) is the generalized Lommel-Wright function with 4 indices ($\mu > 0, q \in \mathbb{N}, \nu, \lambda \in \mathbb{C}$), introduced by de Oteiza, Kalla and Conde [14]:

$$\begin{aligned} J_{\nu,\lambda}^{\mu,q}(z) &= (z/2)^{\nu+2\lambda}\sum_{k=0}^{\infty}\frac{(-1)^k(z/2)^{2k}}{(\Gamma(\lambda+k+1))^q\,\Gamma(\nu+k\mu+\lambda+1)} \\ &= (z/2)^{\nu+2\lambda}E^{(q+1)}_{(\mu,1,...,1),(\nu+\lambda+1,\lambda+1,...,\lambda+1)}\left(-(z/2)^2\right). \end{aligned} \tag{3}$$

This is an interesting example of a multi-index M-L function with arbitrary $m = q+1$. For $m = 2$ the functions (2) are Dzrbashjan's M-L type functions from [1], denoted as $E_{(1/\rho_1,1/\rho_2),(\mu_1,\mu_2)}$.

Obviously for $q = 1$, special function (3) turns into the generalization of the Bessel function $J_\nu(z)$, introduced by Pathak [19] (for details see [11]):

$$J^{\mu}_{\nu,\lambda}(z) = (z/2)^{\nu+2\lambda} \sum_{k=0}^{\infty} \frac{(-1)^k (z/2)^{2k}}{\Gamma(\lambda+k+1)\Gamma(\nu+k\mu+\lambda+1)} \tag{4}$$

$$= (z/2)^{\nu+2\lambda} E^{(2)}_{(\mu,1),(\nu+\lambda+1,\lambda+1)}(-(z/2)^2).$$

For particular choices of the other parameters λ and μ we obtain results for more special cases, e.g., the classical Bessel functions $J_\nu(z)$ and so called Bessel-Maitland function $J^{\mu}_{\nu}(z)$ (on the name E. Maitland Wright), introduced in [21].

SERIES IN MITTAG-LEFFLER TYPE FUNCTIONS

We introduce the following auxiliary functions, related to the Mittag-Leffler functions:

$$\widetilde{E}_{0,\beta}(z) = 1; \quad \widetilde{E}_{n,\beta}(z) = \Gamma(\beta) z^n E_{n,\beta}(z), \tag{5.β}$$

$$\widetilde{E}_{\alpha,0}(z) = 1; \quad \widetilde{E}_{\alpha,n}(z) = \Gamma(n) z^n E_{\alpha,n}(z), \tag{5.α}$$

$$(n \in \mathbb{N}; \quad \beta > 0, \quad \alpha > 0),$$

and consider series in these functions in the complex plane, respectively of the forms:

$$\sum_{n=0}^{\infty} a_n \widetilde{E}_{n,\beta}(z), \quad (6.\beta); \quad \sum_{n=0}^{\infty} a_n \widetilde{E}_{\alpha,n}(z), \quad (6.\alpha); \quad \sum_{n=0}^{\infty} a_n J^{\mu,m}_{n-2\lambda,\,\lambda}(z) \tag{6}$$

$$(m \in \mathbb{N}, \quad \mu > 0, \quad \lambda \in \mathbb{C})$$

with complex coefficients a_n $(n = 0,1,2,...)$.

Our main objective is to study the convergence of the series $(6.\beta)$, $(6.\alpha)$ and (6) in the complex plane. To this end we need suitable asymptotic formulae for the functions $E_{n,\beta}(z)$, $E_{\alpha,n}(z)$ and $J^{\mu,m}_{n-2\lambda,\,\lambda}(z)$ with respect to the "large" values of n.

Here we give such a formula for the functions

$$J^{\mu,m}_{n-2\lambda,\lambda}(z) = (z/2)^n \sum_{k=0}^{\infty} \frac{(-1)^k (z/2)^{2k}}{(\Gamma(\lambda+k+1))^m \Gamma(n-\lambda+k\mu+1)}, \tag{7}$$

whereas for the other considered functions the formulae are given in [18].

Remark 1. Note, that it is possible some coefficients to vanish depending on the values of the parameter λ, that is, to exist numbers $p \in \mathbb{N}_0$, $s \in \mathbb{N}$ such that the identity (7) to take the form

$$J^{\mu,m}_{n-2\lambda,\lambda}(z) = (z/2)^n \left(\frac{(-1)^p (z/2)^{2p}}{(\Gamma(\lambda+p+1))^m \Gamma(n-\lambda+p\mu+1)} \right. \tag{8}$$

$$+\sum_{k=p+s}^{\infty} \frac{(-1)^k (z/2)^{2k}}{(\Gamma(\lambda+k+1))^m \Gamma(n-\lambda+k\mu+1)} \Bigg).$$

Theorem 1. *Let $\mu > 0$. Then the Lommel-Wright functions (7) have the following asymptotic formula*

$$J_{n-2\lambda,\lambda}^{\mu,m}(z) = \frac{(-1)^p (z/2)^{n+2p}}{(\Gamma(\lambda+p+1))^m \Gamma(n-\lambda+p\mu+1)} (1+\theta_{n-2\lambda,\lambda}^{\mu,m}(z)), \tag{9}$$

$$\theta_{n-2\lambda,\lambda}^{\mu,m}(z) \to 0 \ \text{as} \ n \to \infty \quad (n \in \mathbb{N}_0).$$

The convergence on the compact subsets of the complex plane $\mathbb{C}$ is uniform and

$$\theta_{n-2\lambda,\lambda}^{\mu,m}(z) \; = \; O\left(\frac{1}{n^{s\mu}}\right). \tag{10}$$

The Proof will be published elsewhere.

In the next sections we propose theorems, corresponding to the classical Cauchy-Hadamard, Abel, Tauber and Littlewood theorems for power series. Such kind of results are provoked by the fact that the solutions of some fractional order differential and integral equations can be written in terms of series (or series of integrals) of Mittag-Leffler functions. Convergence theorems have also been obtained for series in other special functions, for example, for series in Laguerre and Hermite polynomials by other authors, and resp. by the author — for series in Bessel functions and their Wright's 2, 3, and 4-index generalizations, see the previous papers [15]-[17].

CAUCHY-HADAMARD AND ABEL TYPE THEOREMS

First we give a theorem of Cauchy-Hadamard type for each of the above series.

Theorem 2 (of Cauchy-Hadamard type). *The domain of convergence of each one of the series $(6.\beta)$, $(6.\alpha)$, (6) with complex coefficients a_n is the disk $|z| < R$ with the radius of convergence $R = 1/\Lambda$, where:*

$$\Lambda = \limsup_{n\to\infty} \left(\, |a_n| \, \right)^{1/n} \tag{11}$$

for the series $(6.\beta)$, $(6.\alpha)$ and

$$\Lambda = 2^{-1} \limsup_{n\to\infty} \left(|a_n| \, |(\Gamma(\lambda+p+1))^m \, \Gamma(n-\lambda+p\mu+1)|^{-1} \right)^{1/n} \tag{12}$$

for the series (6) in Lommel-Wright functions. The cases $\Lambda = 0$ and $\Lambda = \infty$ are included in the general case, provided $1/\Lambda$ is supposed ∞, respectively 0.

Let $z_0 \in \mathbb{C}$, $0 < R < \infty$, $|z_0| = R$ and g_φ be an arbitrary angular domain with size $2\varphi < \pi$ and with vertex at the point $z = z_0$, which is symmetric with respect to the straight line defined by the points 0 and z_0.

Theorem 3 (of Abel type). *Let $\{a_n\}_{n=0}^{\infty}$ be a sequence of complex numbers, Λ be defined as in Theorem 2, $0 < \Lambda < \infty$. Let $K = \{z : z \in \mathbb{C}, |z| < R, R = 1/\Lambda\}$.*

If $g(z;\beta)$, $h(z;\alpha)$, $j(z)$ are, respectively, the sums of the series $(6.\beta)$, $(6.\alpha)$, (6) on the domain K, and these series converge at the point z_0 of the boundary of K, then:

$$\lim_{z\to z_0} g(z;\beta)=\sum_{n=0}^{\infty} a_n\widetilde{E}_{n,\,\beta}(z_0), \quad (13.\beta); \qquad \lim_{z\to z_0} h(z;\alpha)=\sum_{n=0}^{\infty} a_n\widetilde{E}_{\alpha,n}(z_0); \quad (13.\alpha)$$

$$\lim_{z\to z_0} j(z)=\sum_{n=0}^{\infty} a_n J^{\mu,m}_{n-2\lambda,\lambda}(z_0), \qquad (13)$$

provided $|z|<R$ and $z\in g_\varphi$.

The Proofs of Theorem 2 and Theorem 3, using the specific properties of the considered special functions, follow the lines of the analogous type theorems in [15]-[18].

(E,z_0) - SUMMATIONS

Let us consider the numerical series

$$\sum_{n=0}^{\infty} a_n, \quad a_n\in\mathbb{C}, \quad n=0,1,2,... \qquad (14)$$

Note that each of the functions $\widetilde{E}_{n,\beta}(z),\widetilde{E}_{\alpha,n}(z),J^{\mu,m}_{n-2\lambda,\lambda}(z)$, $n\in\mathbb{N}$, being an entire function, not identically zero, has at most a finite number of zeros in the closed and bounded set $|z|\le R$. Moreover, due to proven asymptotic formulae, only finite number of these functions may have some zeros at all, except 0.

Let $z_0\in\mathbb{C}$, $|z_0|=R$, $0<R<\infty$, $\widetilde{E}_{n,\beta}(z_0)\neq 0$, $\widetilde{E}_{\alpha,n}(z_0)\neq 0$ and $J^{\mu,m}_{n-2\lambda,\lambda}(z_0)\neq 0$. For the sake of brevity, we denote

$$J^{*}_{n,\lambda,\mu,m}(z;z_0)=\frac{J^{\mu,m}_{n-2\lambda,\lambda}(z)}{J^{\mu,m}_{n-2\lambda,\lambda}(z_0)},\ E^{*}_{n,\beta}(z;z_0)=\frac{\widetilde{E}_{n,\beta}(z)}{\widetilde{E}_{n,\beta}(z_0)},\ \ E^{*}_{\alpha,n}(z;z_0)=\frac{\widetilde{E}_{\alpha,n}(z)}{\widetilde{E}_{\alpha,n}(z_0)}. \qquad (15)$$

Further, we introduce the following Abel type of summability, related to series in M-L functions.

Definition 2. The series (14) is said to be (J,z_0)- summable (respectively (E_β,z_0), (E_α,z_0) - summable), if the series

$$\sum_{n=0}^{\infty} a_n J^{*}_{n,\lambda,\mu,m}(z;z_0), \qquad (16)$$

respectively

$$\sum_{n=0}^{\infty} a_n E^{*}_{n,\beta}(z;z_0), \qquad (16.\beta); \qquad \sum_{n=0}^{\infty} a_n E^{*}_{\alpha,n}(z;z_0), \qquad (16.\alpha)$$

converges in the disk $|z|<R$, and moreover, there exists the limit

$$\lim_{z\to z_0}\sum_{n=0}^{\infty} a_n J^{*}_{n,\lambda,\mu,m}(z;z_0), \qquad (17)$$

respectively

$$\lim_{z\to z_0}\sum_{n=0}^{\infty} a_n E^*_{n,\beta}(z;z_0), \qquad (17.\beta); \qquad\qquad \lim_{z\to z_0}\sum_{n=0}^{\infty} a_n E^*_{\alpha,n}(z;z_0), \qquad (17.\alpha)$$

provided z remains on the segment $[0,z_0]$.

Remark 2. Every (J,z_0), $(E_\beta,z_0),(E_\alpha,z_0)$, - summation is regular, and this property is just a particular case of Theorem 3.

TAUBERIAN TYPE THEOREMS

A Tauberian theorem is a statement that relates the Abel summability and the standard convergence of a numerical series by means of some assumptions imposed on the general term of the series under question. A classical result in this direction is given by Theorem 85 in Hardy [6].

In this paper we extend the validity of such type of assertion to series in Mittag-Leffler and Lommel-Wright functions, by means of the following theorem.

Theorem 4 (of Tauber type). *If the series* (14) *is* (J,z_0)*- summable (resp.* (E_β,z_0) (E_α,z_0)*), and*

$$\lim_{n\to\infty} na_n = 0, \tag{18}$$

then it is convergent.

Tauber type theorems have been given also for summations by means of Bessel type functions by the author in [15], [17].

At first sight, it seems that the condition $a_n = o(1/n)$ is essential. Nevertheless, Littlewood succeeded to weaken it and to obtain the stronger version of the Tauber theorem (see [6], Theorem 90).

A Littlewood generalization of the $o(n)$ - version of the Tauber type theorem (Theorem 4) is given below. Similar theorems have been proved in [17] and [18].

Theorem 5 (of Littlewood type). *If the series (14) is* (J,z_0) *- summable, resp.* (E_β,z_0)*,* (E_α,z_0) *- summable, and*

$$a_n = O(1/n), \tag{19}$$

then the series (14) converges.

Proof. Let z belong to the segment $[0,z_0]$. Using the asymptotic formula (9) for the Lommel-Wright functions, we obtain:

$$a_n J^*_{n,\lambda,\mu,m}(z;z_0) = a_n\left(\frac{z}{z_0}\right)^{n+2p}\frac{1+\theta^{\mu,m}_{n-2\lambda,\lambda}(z)}{1+\theta^{\mu,m}_{n-2\lambda,\lambda}(z_0)} = a_n\left(\frac{z}{z_0}\right)^{n+2p}\left(1+\widetilde{\theta}^*_{n,\lambda,\mu,m}(z;z_0)\right),$$

where $\widetilde{\theta}^*_{n,\lambda,\mu,m}(z;z_0) = \dfrac{\theta^{\mu,m}_{n-2\lambda,\lambda}(z)-\theta^{\mu,m}_{n-2\lambda,\lambda}(z_0)}{1+\theta^{\mu,m}_{n-2\lambda,\lambda}(z_0)}$. Then $\widetilde{\theta}^*_{n,\lambda,\mu,m}(z;z_0) = O(1/n^{s\mu})$, due to (10).

Let us write (16) in the form

$$\sum_{n=0}^{\infty} a_n J^*_{n,\lambda,\mu,m}(z;z_0) = \sum_{n=0}^{\infty} a_n \left(\frac{z}{z_0}\right)^{n+2p} \left(1+\widetilde{\theta}^*_{n,\lambda,\mu,m}(z;z_0)\right). \tag{20}$$

Denoting $w_n(z) = a_n \left(\frac{z}{z_0}\right)^{n+2p} \widetilde{\theta}^*_{n,\lambda,\mu,m}(z;z_0)$, we consider the series $\sum\limits_{n=0}^{\infty} w_n(z)$. Since $|w_n(z)| \le |a_n|\,|\widetilde{\theta}^*_{n,\lambda,\mu,m}(z;z_0)|$ and according to condition (10) and (19), there exists a constant $\widetilde{C}$, such that $|w_n(z)| \le \widetilde{C}/n^{1+s\mu}$ for $n \in \mathbb{N}$. Since $\sum\limits_{n=1}^{\infty} 1/n^{1+s\mu}$ converges, the series $\sum\limits_{n=0}^{\infty} w_n(z)$ is also convergent, even absolutely and uniformly on the segment $[0,z_0]$. Therefore, since $\lim\limits_{z\to z_0} w_n(z) = w_n(z_0) = a_n\widetilde{\theta}^*_{n,\lambda,\mu,m}(z_0;z_0) = 0$, then

$$\lim_{z\to z_0} \sum_{n=0}^{\infty} w_n(z) = \sum_{n=0}^{\infty} \lim_{z\to z_0} w_n(z) = 0.$$

Obviously, the assumption that the series (14) is $(J,z_0;\mu)$ – summable implies the existence of the limit (17). Then, having in mind that (20) can be written in the form

$$\sum_{n=0}^{\infty} a_n J^*_{n,\lambda,\mu,m}(z;z_0) = \sum_{n=0}^{\infty} a_n \left(\frac{z}{z_0}\right)^{n+2p} + \sum_{n=0}^{\infty} a_n \left(\frac{z}{z_0}\right)^{n+2p} \widetilde{\theta}^*_{n,\lambda,\mu,m}(z;z_0),$$

we conclude that there exists the limit

$$\lim_{z\to z_0} \sum_{n=0}^{\infty} a_n \left(\frac{z}{z_0}\right)^{n+2p} \tag{21}$$

and, moreover,

$$\lim_{z\to z_0} \sum_{n=0}^{\infty} a_n J^*_{n,\lambda,\mu,m}(z;z_0) = \lim_{z\to z_0} \sum_{n=0}^{\infty} a_n \left(\frac{z}{z_0}\right)^{n+2p}.$$

From the existence of the limit (21) it follows that the series (14) is A-summable. Then, according to Theorem 90 ([6]), the series (14) converges.

Using the asymptotic formulae and estimates for the M-L functions, given in [18], the proofs of the other two cases go analogously. ∎

ACKNOWLEDGMENTS

This paper is supported by the Project ID 02/25/2009 “ITMSFA” of the National Science Fund - Ministry of Education, Youth and Science of Bulgaria.

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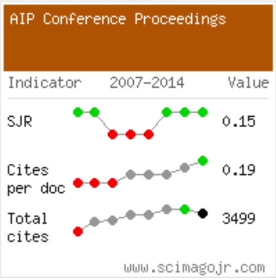

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